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MODIFIED HYPERBOLIC STRESS～STRAIN RESPONSE: UNCEMENTED AND CEMENT STABILIZED CLAYS

By

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ABSTRACT : It is revealed that the cement stabilized clays exhibit strain softening behavior due to the destruction of the samples after peak deviator stress (Horpibulsuk et al., 2000). On the other hand, the uncemented clays would not show such behavior because only the friction between grains controls the strength characteristic. A simple model based on the hyperbolic function is proposed herein to capture the undrained and drained behavior both of uncemented and cement stabilized clays. The advantage of this model is that close agreement between experimental and predicted responses is achieved with a reasonable degree of accuracy and the parameters for the analysis are simply determined from the conventional triaxial tests.

INTRODUCTION

It has been bought out that the factors controlling the strength characteristics of cement stabilized clays are friction between grains (fabric) and cementation bond by Horpibulsuk et al., 2000. The cementation bond is broken down at the peak strength, leading to the strain softening. On the other hand, the uncemented clay would not exhibit such behavior because the strength characteristics are controlled by only friction between grains. As a result, the behavior of uncemented clay is simpler to assess than that of cement stabilized clay as evident by many numerical models such as Cam clay model, modified Cam clay model and so on.

The advantage of the Cam clay and modified Cam clay models is that a few parameters for the analysis are required and can be simply obtained for the standard tests (oedometer and traixal tests).

Vitasala (1989) hypothesized that the load carrying capacity of the cemented soils can be spilt into two components ; namely, uncemented and cementation bond. The deformation of soil is essentially due to change in stress increments on equivalent unbonded soil skeleton.

A rate-independent constitutive model for natural clays was formulated within the framework of kinematic hardening with elements of bounding surface plasticity by Rouainia and Muir Wood (2000). This model is an extension from the Cam clay model.

Kasama et al. (2000) predicted the stress strain behavior of lightly cemented clay based on the extended critical state concept. However, their method cannot explain the behavior of highly cemented clay since the strain softening behavior is not taken into account by their method.

Most of the constitutive variables involved in these proposed models require elaborate experi-

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mental program. An attempt to make a simple model to capture the undrained and drained behavior of natural clays and cement stabilized clays has been done in this investigation. The simple model based on the hyperbolic function is proposed herein. The hyperbolic model has been successfully done for sensitive clays (Nagendra Prasad et al., 1999). The modified hyperbolic model has been introduced to predict the undrained stress～strain response of the naturally cemented Ariake clay as well as cement stabilized Ariake clay (Horpibulsuk and Miura, 2001).

APPLICATION OF MODIFIED HYPERBOLIC MODEL

The modified hyperbolic model is proposed to predict the stress～strain response of the cement stabilized clay under undrained and drained shear in this section for the sake of the simple analysis. The behavior of cement stabilized samples is different from that of uncemented samples. Strain softening is observed under effective cell pressures even far higher than the apparent yield stress. Most important difference is that the softening is associated with positive pore pressure in undrained shear and with positive volumetric strain in drained shear whereas the same would not happen for uncemented samples. As a result, the numerical models to capture the behavior of the cement stabilized clays are complicated and still under researching. The aim of this paper is to present the simple practical procedure for representing the softening behavior of cement stabilized clay during undrained and drained shear.

UNDRAINED BEHAVIOR

An examination of the data obtained from experimental results (Horpibulsuk et al., 2000 and Horpibulsuk, 2001) shows that the mean effective stress～shear strain (p', ε_S) (*vide* Figures 1 and 2) relations of the uncemented samples and the cement stabilized samples at low cement content of 6 % are hyperbola, whereas it is initially hyperbola and softens after the peak for the cement stabilized samples of 9 % cement (*vide* Figure 3). The stress ratio～shear strain (η, ε_S) (*vide* Figure 4)

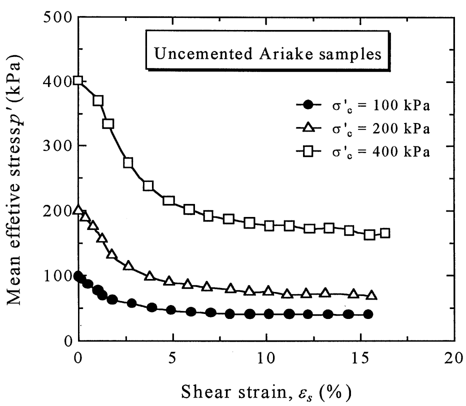

Figure 1. Mean effective stress～shear strain relationships of uncemented Ariake clay

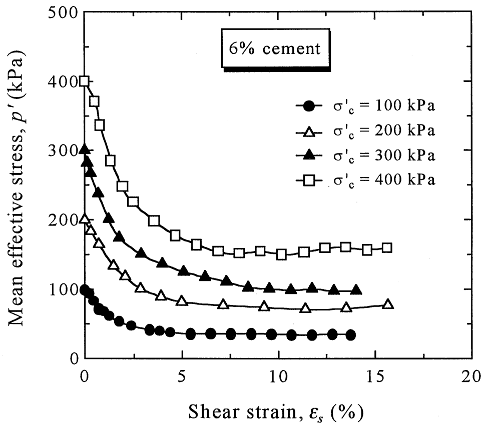

Figure 2. Mean effective stress～shear strain relationships of 6% cement samples

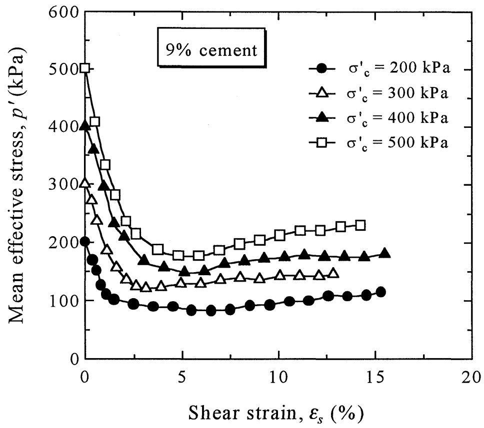

Figure 3. Mean effective stress～shear strain relationships of 9% cement samples

relation of uncemented sample is hyperbola while it shows the hyperbolic relationship with softening behavior (*vide* Figures 5 and 6) for all cement stabilized samples. As a result, the modified hyperbolic model is required in this analysis. The analysis based on the (p', ε_S) and (η, ε_S) relations is more advantage because the hyperbolic relationship is recognized in case of uncemented, naturally cemented and lightly cemented samples, whereas the (q, ε_S) and (Δu, ε_S) relations of the all cemented samples are strain softening. The variation of the stress ratio and the mean effective stress with the shear strain in terms of modified hyperbolic relation takes the form as

$$\eta = \frac{\varepsilon_S}{a_1 \; b_1 \; \varepsilon_S^{n_1}} \tag{1}$$

$$(p_0' - p') = \frac{\varepsilon_S}{a_2 + b_2 \; \varepsilon_S^{n_2}} \tag{2}$$

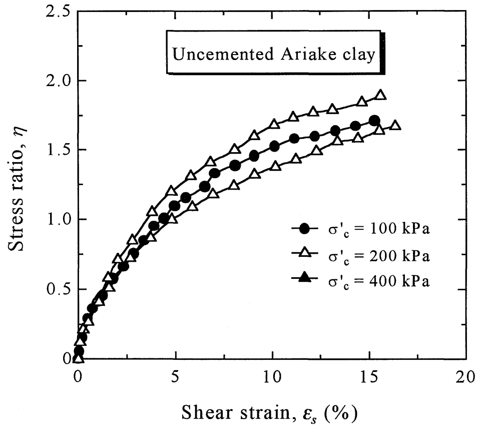

Figure 4. Stress ratio～shear strain relationships of uncemented Ariake clay

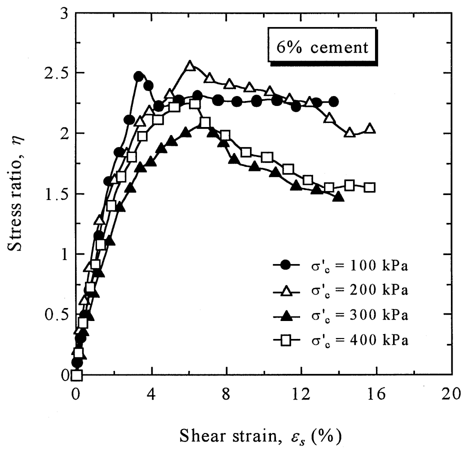

Figure 5. Stress ratio～shear strain relationships of 6% cement samples

where p_0' is the initial effective mean stress (kPa), ε_S is the shear strain (%), a_1, b_1, n_1, a_2, b_2 and n_2 are constants. As a specific case when $n_1 = n_2 = 1.0$, these equations reduce to the one suggested by Kondner (1963). The shape of the curve is hyperbola without any marked yield point and the peak is reached as asymptotically at finite strain. The process to obtain these parameters is presented in Appendix. For meaningful application of the relations proposed, it is preferable to determine all the parameters in terms of the initial effective cell (mean effective) pressure so as to simply apply in the computer analysis.

It is found that the hyperbolic model ($n_1 = n_2 = 1.0$) can be well applied for the uncemented samples as shown in Figure 7. This application is very useful for the uncemented clay because the stress ratio～shear strain relationships (η, ε_S) of all samples are almost in the same feature, that the same set of parameters ($a_1 = 1.932$ and $b_1 = 0.474$) can be applied. The parameters a_2 and b_2 are

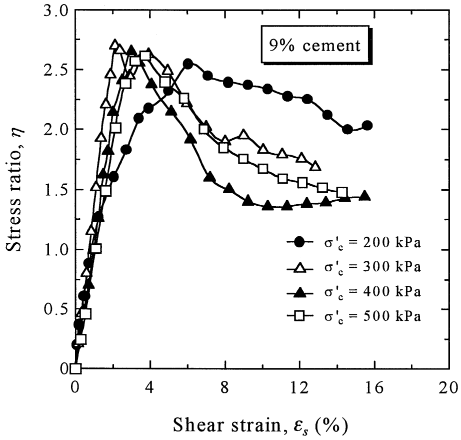

Figure 6. Stress ratio～shear strain relationships of 9% cement samples

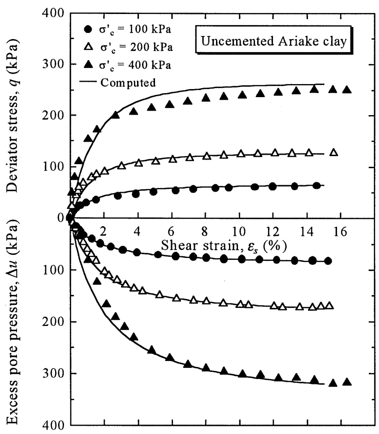

Figure 7. Calculated and Experimental deviator stress and excess pore pressure versus shear strain curves for uncemented Ariake clay

presented in terms of initial effective cell pressures in a form of a power function as follows.

$a_2 = 0.572\,(p_0')^{-0.706}$ $\quad R^2 = 0.943$ (3)

$b_2 = 1.438\,(p_0')^{-1.000}$ $\quad R^2 = 0.996$ (4)

Figure 8 shows the computed and experimental relationships of the cement stabilized Ariake clay at cement content of 6%. As the hyperbolic model can be applied to the (p', ε_S) relationship, n_2 is taken as unity. The modified hyperbolic relation is applied to the (η, ε_S) relationship with n_1 of 2.0 and b_1 of 0.004. The a_1 is expressed in the form of exponential function in terms of the initial effective cell pressure. The parameters are shown as follows

$a_1 = 0.641\,\exp\,(0.002p_0')$ $\quad R^2=0.965$ (5)

$a_2 = 0.145\,(p_0')^{-0.579}$ $\quad R^2=0.909$ (6)

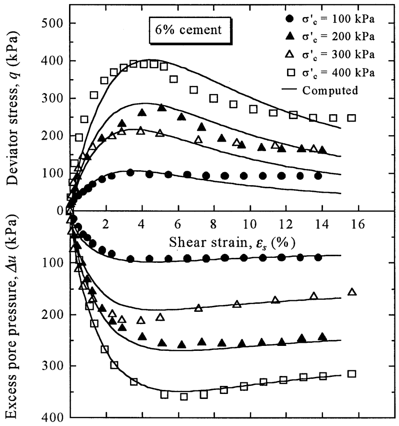

Figure 8. Calculated and Experimental deviator stress and excess pore pressure versus shear strain curves for 6% cement samples

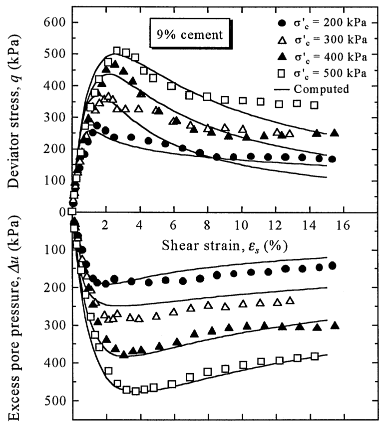

Figure 9. Calculated and Experimental deviator stress and excess pore pressure versus shear strain curves for 9% cement samples

$b_2 = 1.582\,(p'_0)^{-1.103}$ $R^2 = 0.998$ (7)

The analysis of stress～strain curves of the cement stabilized clay at 9% cement subjected to the effective cell pressures higher than its apparent yield stress as shown in Figure 9.

The parameters for the analysis are presented as follows

$a_1 = 0.215 \exp (0.002 p'_0)$ $R^2 = 0.940$ (8)

$b_1 = 0.127 - 0.002\,(p'_0)$ $R^2 = 0.948$ (9)

$(n_1 = 2.0)$

$a_2 = 0.105\,(p'_0)^{-0.548}$ $R^2 = 0.980$ (10)

$b_2 = 0.430\,(p'_0)^{-1.269}$ $R^2 = 0.997$ (11)

$(n_2 = 1.25)$

DRAINED BEHAVIOR

It is desirable to predict the drained behavior from the (q, ε_S) and (ε_ν , ε_S) relations directly, unlike the undrained behavior because the simple hyperbolic model can be widely used. The equations proposed are as follows

$$q = \frac{\varepsilon_S}{a_3 + b_3\ \varepsilon_S^{n3}} \tag{13}$$

$$\varepsilon_\nu = \frac{\varepsilon_S}{a_4 + b_4\ \varepsilon_S^{n4}} \tag{14}$$

where the units of q, ε_S and ε_ν are kPa, percentage and percentage, respectively.

This method of analysis merits predicting the drained behavior of the uncemented clay and the cement stabilized clay, which are subjected to the effective cell pressures higher than the apparent yield stress because the (ε_ν , ε_S) relation is unique.

For the uncemented clay (*vide* Figure 10), the parameters n_3 and n_4 are taken as one. The parameters for (ε_ν , ε_S) relation, a_4 and b_4 are 0.810 and 0.041, respectively which do not change with the change in effective cell pressures. The variation of parameters a_3 and b_3 with initial mean effective stress are expressed in the form of power function.

$$a_3 = 2.907(p_0')^{-0.915} \qquad R^2 = 0.996 \tag{15}$$

$$b_3 = 0.015(p_0')^{-0.511} \qquad R^2 = 0.996 \tag{16}$$

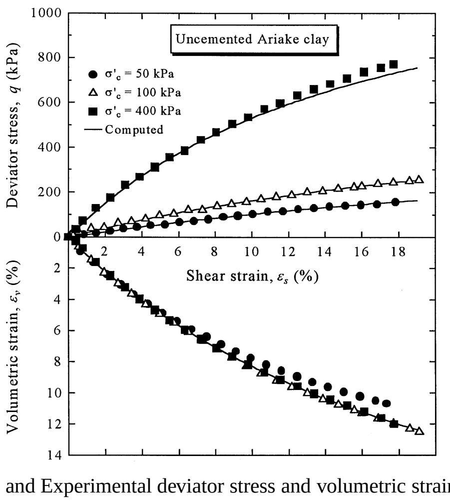

Figure 10.Calculated and Experimental deviator stress and volumetric strain versus shear strain curves for uncemented Ariake clay

The calculated curves comparing with the experimental curves of the cement stabilized Ariake clay samples at low cement content of 6% are shown in Figure 11. It is found that the hyperbolic model can be applied well with these curves as well ; hence, the n_3 = n_4 =1.0. The a_4 and b_4 are taken as 0.552 and 0.020, respectively for all effective cell pressures. The a_3 and b_3 are expressed as follows.

$$a_3 = 0.055 \exp(-0.0045 p'_0) \qquad R^2 = 0.992 \tag{17}$$

$$b_3 = 0.012 (p'_0)^{-0.693} \qquad R^2 = 0.932 \tag{18}$$

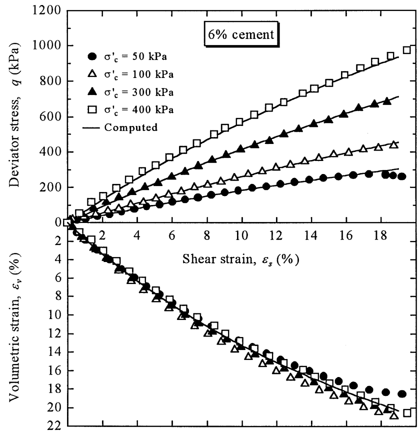

Figure 11 Calculated and Experimental deviator stress and volumetric strain versus shear strain curves for 6% cement samples

CONCLUSION

This paper aims to present a simple numerical form to represent the behavior of the uncemented and the cement stabilized clays under undrained and drained conditions within the framework of modified hyperbolic stress～strain response. It is found that the calculated and experimental curves are in good agreement. This proposed method is useful for capturing the undrained and drained behavior of not only uncemented clays but also the cement stabilized clays that the strain softening behavior is realized.

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APPENDIX

The process to obtain the parameters for modified hyperbolic model is presented herein.

$$\eta = \frac{\varepsilon_S}{a_1 + b_2\, \varepsilon_S^{n_1}} \tag{A 1}$$

The strain at maximum hmax (failure strain), ε_{sf} is reached when $\mathrm{d}\eta/\mathrm{d}\varepsilon_S = 0$

$$\left\{\frac{d\eta}{d\varepsilon_S}\right\} = \frac{(a_1 + b_1\, \varepsilon_S^{n_1}) - \varepsilon_S\, b_1\, n_1\, \varepsilon_S^{n1}}{(a_1 + b_1\, \varepsilon^{n_1})^2} = 0 \tag{A 2}$$

$$(a_1\, b_1\, \varepsilon_{Sf}^{n1}) - n_1\, b_1\, \varepsilon_{Sf}^{n1} = 0 \tag{A 3}$$

Thus,

$$\varepsilon_{Sf} = \left\{\frac{a_1}{b_1\, (n_1 - 1)}\right\}^{\frac{1}{n_1}} \tag{A 4}$$

$$\eta_{\max} = \frac{\varepsilon_{Sf}\,(n_1 - 1)}{a_1\, n_1} \tag{A 5}$$

From A (4) and A (5) the a_1 and b_1 can be obtained in terms of ε_{Sf}, $\eta_{\max}$ and n_1 as follows.

$$a_1 = \frac{\varepsilon_{Sf}}{\eta_{\max}}\left\{\frac{n_1 - 1}{n_1}\right\} \tag{A 6}$$

$$b_1 = \frac{a_1}{(n_1 - 1)\,\varepsilon_{Sf}^{n1}} \tag{A 7}$$

Introducing the values of a_1, $b_1$and n_1, the modified hyperboli relation would be

$$\frac{\eta_\varepsilon}{\eta_{\max}} = \frac{n_1\, \frac{\varepsilon_S}{\varepsilon_{Sf}}}{(n_1 - 1) + \left(\frac{\varepsilon_S}{\varepsilon_{Sf}}\right)} \tag{A 8}$$

At a specific case, the value of $\frac{\varepsilon}{\varepsilon_{Sf}}$ is taken as 0.5 i.e., the ratio of strain at failure to half strain as the basis for determining the value of n_1.The $a_1$and $b_1$can be then attained by A(6) and A(7).

The other parameters can be acquired by the same process.